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Using Bounded Rationality to Improve Decentralized Design

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The design of large scale complex engineering systems requires interaction and communication between multiple disciplines and decentralized subsystems. Game theory has been used previously to model interactions between distributed multidisciplinary design subsystems and predict convergence and equilibrium solutions. These game theoretic models assume that designers make perfectly rational decisions by selecting solutions from their *rational reaction set*. For convergent decentralized design problems, the intersection of the designers' rational reaction set results in a Nash equilibrium solution where designers may converge if they were making rational choices. However, this equilibrium solution is rarely optimal from a multi-objective optimization perspective. Further, empirical studies reject the claim that decision makers always make rational choices and the concept of *bounded rationality* is used to explain such behavior. In this paper, a framework is proposed that uses the idea of bounded rationality in conjunction with set-based design, metamodeling, and multi-objective optimization techniques to improve solutions for convergent decentralized design problems. Through the use of this framework convergent decentralized design problems converge to solutions that are superior to the Nash equilibrium.

I. Introduction

THE design of large scale complex systems such as aircraft, space systems, and automobiles requires input from several different disciplines or subsystems. When these disciplines or subsystems are represented by geographically distributed design teams or a set of independent suppliers, what results is a decentralized design process. There exist many challenges in the organization and design of such decentralized systems. The study of developing optimal design processes and configurations for large scale multidisciplinary systems is called multidisciplinary design optimization (MDO) [1,2]. Initially focused on the design of aircraft, MDO methodology and research has now expanded to the design of all complex systems. In this paper, the underlying assumption of rational decision making in MDO problems is investigated. Lessons learned from this investigation are incorporated into a new method for solving decentralized design and MDO problems. This new method seeks to improve over solutions obtained from traditional solution approaches by relaxing the prescriptive assumption of rationality in the solution process and studying the existence of normative bounded rationality

There exist several frameworks and methodologies developed for formulating and solving MDO problems [3]. One of the earliest developed methodologies was collaborative optimization (CO). The fundamental concept in CO required the system-level designer (optimizer) to provide targets to subsystems. These targets optimize the system-level objective function, and subsystems determine designs that minimize the difference between the current states and targets provided by the system-level designers. Also, subsets of the system-level constraints are assigned to the most closely associated subsystem disciplines [1,4–6]. The bilevel integrated systems synthesis is very similar to CO as a framework for solving MDO problems [7–9], where the overall MDO problem is divided into smaller optimizations performed at the system and subsystem levels. Approximate mathematical models are used to transfer information from the subsystem optimizations to the system optimization.

In the concurrent subspace optimization (CSSO) method, the system-level designer performs no optimization and only serves as a coordinator between subsystems. The primary responsibility of the system-level designer is to ensure feasibility of system-level constraints. One of the drawbacks of CSSO is the difficulty in achieving convergence due to the lack of coordination strategy at the system level to arbitrate among subsystem discrepancies [10–14]. More recently, the analytical target cascading (ATC) method has been developed for MDO. ATC is similar to CO in that the system-level designer passes targets to the subsystem level. However, ATC allows for multiple hierarchical levels of decomposition as opposed to the bilevel approach stated in CO. The targets within ATC cascade through the multiple levels of hierarchy. The approach prescribed within ATC bears closer resemblance to real world normative design scenarios, where multiple subsystems and suppliers form a multilevel hierarchical product design structure. More important, the convergence properties of ATC represent a significant advantage [15,16].

In addition to the aforementioned methods, there exist several other approaches that have been developed for solving MDO problems. Two such methods are EMDO [17], where the MDO process is viewed as a traditional design process but with increased used of optimization at every juncture, and the collaboration pursuing method [18,19], which is a sampling based approach that does not require sensitivity analysis.

A common characteristic of these frameworks is that all subsystems work concurrently and report to some higher level (sub) system. Contrary to these methods, this paper uses a different approach for decentralized product design where there exists no "system-level" designer; rather, the individual subsystems communicate directly with each other iteratively exchanging design information. The sequential iterative approach shown in Fig. 1 requires the subsystems to individually make their decisions in a sequence, communicate their decisions to the next discipline, and

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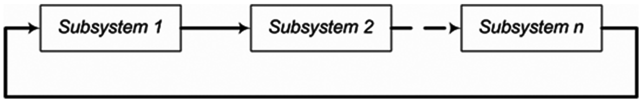

Fig. 1 Schematic of the sequential iterative approach.

iterate until convergence [20–24]. Convergence is obtained when all disciplines or subsystems reach a solution that cannot be improved by further iteration.

The important distinction between the sequential iterative approach used as a baseline in this paper and the various MDO approaches described earlier is the information being communicated between the designers. Although the MDO approaches advocate communication of various combinations of design variables, objective functions, targets, gradients, and constraint information, the sequential iterative approach only requires communication of design variables. This is significant because corporate secrecy and globally distributed product design may prevent the various subsystems from extensive sharing of more comprehensive design information between the subsystems. Having simplified information sharing protocols is even more important where the individual subsystems are suppliers/subcontractors that may be competitors as well. Because of the interactive, negotiation based nature of the sequential iterative approach to design large scale systems, concepts from game theory are a natural choice as a modeling tool [25]. Additionally, the form of communication between subsystems within the sequential iterative process, where the only communicated information are design variable values, is functionally the same as the noncooperative protocol in game theory terminology.

In addition to game theory, the framework developed in this paper builds upon several different concepts, such as multi-objective optimization and bounded rationality. These concepts are presented in the next section.

II. Background

A. Game Theory in Decentralized Design

The standard sequential iterative approach used for distributed design problems can capitalize on using game theoretic formulations for modeling the interaction between subsystems. Game theory provides mathematical protocols for determining decision solutions where multiple components (subsystems) are involved in the (design) decision-making process. In game theory terminology, the disciplines or design teams are referred to as players and the converged solution within the noncooperative environment is called the Nash equilibrium. In a noncooperative environment decision makers do not share objective function, gradient, nor constraint information. Additionally, in a sequential iterative process, it is assumed that all design teams are making a rational decision. The aggregate of each subsystem's rational decisions is called the rational reaction set (RRS) [26–29].

To mathematically represent the Nash solution, consider two disciplines, subsystem 1 and subsystem 2, each minimizing objective functions f_1 and f_2, respectively. Subsystem 1 minimizes x_1, while subsystem 2 minimizes x_2. The solution (x_1^N, x_2^N) is a Nash solution if

$$f_1\left(x_1^N, x_2^N\right) = \min f_1\left(x_1, x_2^N\right) \qquad x_1 \in X_1$$

and

$$f_2\left(x_1^N, x_2^N\right) = \min f_2\left(x_1^N, x_2\right) \qquad x_2 \in X_2 \tag{1}$$

where X_1 and X_2 are the set of all feasible solutions for x_1 and x_2, respectively. The Nash solution belongs to the following nonlinear map:

$$\left(x_1^N, x_2^N\right) \in X_1^N\left(x_2^N\right) \times X_2^N\left(x_1^N\right) \tag{2}$$

where

$$X_1^N(x_2)\colon \left\{x_1^N \in X_1 \colon f_1\left(x_1^N, x_2\right) = \min f_1(x_1, x_2), x_1 \in X_1\right\} \tag{3}$$

and

$$X_2^N(x_1)\colon \left\{x_2^N \in X_2 \colon f_2\left(x_1^N, x_2\right) = \min f_2(x_1, x_2), x_2 \in X_2\right\} \tag{4}$$

Equations (3) and (4) are the general mathematical representation of the RRS.

Considering the assumption that the decision makers adapt a sequential, iterative approach to solving the design problem, a question that might arise is, why is a sequential iterative approach adapted over a simultaneous or concurrent decision-making approach between the individual subsystems? In previous work, it has been shown that when decision makers are operating either sequentially, simultaneously, or a hybrid of the two, if the individual subsystems continue to make rational decisions (that is, select designs from their RRS) and the information communicated does not change, all processes will either converge to an approximation of the Nash equilibrium or will diverge [27]. Because the goal of this paper is to improve upon Nash equilibrium solutions, either the sequential or simultaneous process can be adapted as the operating framework. Although the sequential approach was selected for this paper, the methodology developed in this paper can be transferred to a simultaneous approach for subsystem communication as well.

The individual subsystems converge to the Nash equilibrium solution given in Eq. (1) when they do not cooperate with each other. When the players work collectively or in collaboration, they are said to be following the cooperative protocol. In such a scenario, subsystems or disciplines can share objective functions, gradients, and constraints information. However, the objectives of the individual subsystems still conflict with each other. Such a problem is referred to as a multi-objective optimization problem. Although this paper focuses on noncooperative, distributed design problems, the formulation and solution of multi-objective optimization problems is used within the developed framework. Therefore, a brief discussion of the multi-objective optimization concepts used in this work is provided in the next section.

B. Multi-Objective Optimization

Multi-objective optimization includes a set of formal tools aimed at providing designers with accurate, complete, and rational information to make effective decisions in engineering. Fundamental to multi-objective optimization in general and to this paper in particular is the concept of Pareto optimality. When multiple competing objectives or criteria exist, the optimum is no longer a single design point but an entire set of nondominated design points. This is commonly known as the Pareto set [30,31]. The Pareto set is composed of Pareto optimal solutions. In simple terms, a Pareto optimal solution is one for which any improvement in one objective must result in the degradation of at least one other objective. Mathematically, a feasible design variable vector, $\bar{x}'$ is Pareto optimal if and only if there is no feasible design variable vector $\bar{x}$, with the characteristics,

$$\begin{aligned} f_i(\bar{x}) &\le f_i(\bar{x}') \quad \text{for all } i, i = 1, \ldots, n \\ f_i(\bar{x}) &< f_i(\bar{x}') \quad \text{for at least one } i, i = 1, \ldots, n \end{aligned} \tag{5}$$

where n is the number of objectives, and the use of the less than symbol indicates an improvement in the objective function (assuming minimization of objectives is desired). Pareto optimality and multi-objective optimization are critical concepts in the development of the framework proposed in this paper.

The fundamental problem in existing solutions of noncooperative, distributed design problems (i.e., the Nash equilibrium) is that it is typically inferior to the cooperative solution (i.e., the Pareto set) [26,27]. The primary assumption that results in the Nash equilibrium solution is that the converged solution is obtained from rational decision making. The goal of this paper is to move solutions from the Nash equilibrium to the Pareto optimal set without the information overload of cooperative protocols, that is, requiring engineers to share objective, constraints, and gradient information. In the next section, the prescriptive assumption of rational decision making is discussed and the notion of bounded rationality is introduced.

C. Rational Decisions and Bounded Rationality

Decision-making models based on game theory and optimization make a fundamental assumption that all decisions are perfectly rational. That is, the decisions made are such that the selected choices are the only options that provide the maximum benefit. However, in previous work, it has been shown that the assumption that designers make rational choices might not always be valid [32]. Empirical evidence is available to conform to the fact that decision makers make mistakes in stating their preferences or choices, even for simple pairwise comparison decisions [33]. Simon coined the phrase *bounded rationality* to explain these mistakes and errors made by humans in decision-making problems [34]. Simon stated that the entire premise of neoclassical economics is based on the foundation of human beings making decisions with the ability to precisely state optimal choices such as those that maximize expected utility. However, based on obtained empirical data, Simon stated that this assumption was not accurate and listed several causes for bounds on human ability to precisely exhibit prescribed rational behavior. Some of these causes are as follows [34]: 1) a limited ability to adjudicate among multiple goals; 2) deficiency in knowledge of exact decision consequences; 3) a limited capability for calculations; 4) the lack of precise decision problem formulation; and 5) the lack of time/computational resources to make a decision. Although Simon studied decision mistakes within economic models, the causes for bounded rationality hold for engineering design decisions as well.

Engineering design has been accepted as a decision-making process and the foundation developed in neoclassical economics has been embraced by engineers. Expected utility theory is a popular normative theory which is also used in engineering design with rational choices being those that provide the highest utility [35–39]. The causes for boundedness in rationality as proposed by Simon are experienced in engineering decision making as well, where engineers in decentralized design environments make decisions with limited available information, managers are required to make decisions with a large number of information inputs, and engineers are constantly working under the pressures of time.

In this paper, the concept of bounded rationality is implemented within decentralized design problems. In previous work, it has been shown that the inclusion of bounded rationality in decentralized design decisions can result in convergent solutions moving from the Nash equilibrium toward Pareto optimality and divergent problems moving toward convergence [40,41]. This work is extended in this paper where a rigorous methodical framework is developed for the improvement of decentralized design solutions through the incorporation of bounded rational decision models. This framework, entitled modified approximation-based decentralized design (MADD), is presented in Sec. III. Section IV presents a two-subsystem case study to demonstrate the proposed framework. Simulation studies to determine the impact of parameters set within the framework are also presented in Sec. IV. Concluding remarks and areas for future work are provided in Sec. V.

III. Modified Approximation-Based Decentralized Design Framework

In the previous section, the sequential iterative process is introduced as a model for the decentralized design of complex engineering systems. The fundamental assumption of the sequential iterative process is that designers communicate locally controlled design variables that are coupled with nonlocal subsystems. No local objective function information is communicated because that information is propriety (noncooperation). These assumptions are retained for the MADD framework which is presented in this section. However, unlike the traditional sequential iterative process, the MADD framework incorporates a set-based design philosophy, where during each iteration more than one "instance" of each design variable is communicated. This vector of design variable values serves as the model for bounded rational choices of the subsystems.

The central idea behind the MADD framework is for each subsystem to approximate nonlocal subsystem objective functions and solve a "local" multi-objective optimization problem. The multi-objective optimization problem consists of the local objective function and the approximated nonlocal objective function. To approximate the nonlocal objective functions, multiple instances of each design variable are communicated between the various subsystems and used within a unique metamodeling technique. The communicated design variable vectors are used to metamodel the rational reaction set of the nonlocal subsystems. These metamodels are then mathematically integrated to approximate the nonlocal objective functions. The idea of integration comes from understanding that for unconstrained objective functions, the RRS is obtained by taking the derivative of the objective with respect to the local design variables. Therefore, an integration of the RRS would result in a form of the objective function. This process is explained with an example as follows.

Consider a design problem with two subsystems, A and B. Each subsystem has two locally controlled design variables, represented as x_1, x_2 for subsystem A and y_1, y_2 for subsystem B. Let the nonlocal objective function of subsystem A (that subsystem B seeks to approximate) at the ith iteration be of the form shown in Eq. (6):

$$F_i^A = x_1^2 + x_2^2 + ax_1 + bx_2 + cx_1y_1 + dx_1y_2 + ex_2y_1 + fx_2y_2 \tag{6}$$

The derivative of F_i^A with respect to x_1 and x_2 (the design variables local to subsystem A) are shown in Eq. (7):

$$\frac{\partial F_i^A}{x_1} = 2x_1 + a + cy_1 + dy_2 \qquad \frac{\partial F_i^A}{x_2} = 2x_2 + b + ey_1 + fy_2 \tag{7}$$

For an unconstrained optimization problem, the necessary condition for optimality is for the gradient to be equal to zero. The result of setting the derivatives of Eq. (7) to zero is shown in Eq. (8):

$$x_1 = -\frac{a}{2} - \frac{c}{2}y_1 - \frac{d}{2}y_2 \qquad x_2 = -\frac{b}{2} - \frac{e}{2}y_1 - \frac{f}{2}y_2 \tag{8}$$

It is noted that Eq. (8) is only a function of the design variables of the two subsystems. Moreover, Eq. (8) represents the rational reaction set of subsystem A if subsystem A's objective function is as defined in Eq. (6).

The motivation behind the MADD framework is to enable decentralized subsystems to "cooperate" without communicating objective function information. In an iterative process, if the two subsystems A and B communicate multiple instances of their individual design variables, it is possible for subsystem B to determine the values for a, b, c, d, e, and f in Eq. (8) using only the communicated design variable vectors (using, for example, response surface techniques). These coefficients can then be used to approximate the objective function for subsystem A in the form shown in Eq. (6).

With the fitting coefficients of Eq. (8) determined, the terms of Eq. (8) can be rearranged to obtain Eq. (7). Upon integration of Eq. (7) and summation of the resulting terms, approximations for the nonlocal objective functions of the form shown in Eq. (6) can be obtained. The process of approximating nonlocal RRS by communicating multiple instances of design variable values, integrating the approximated RRS, and combining them to determine approximations of nonlocal objective functions is the core concept of the MADD framework. The approximated nonlocal objective function is combined with the local objective function to solve a local multi-objective optimization problem.

This proposed approach is advantageous because the solution of the local multi-objective optimization problem introduces "pseudo-cooperation" between subsystems with conflicting objectives

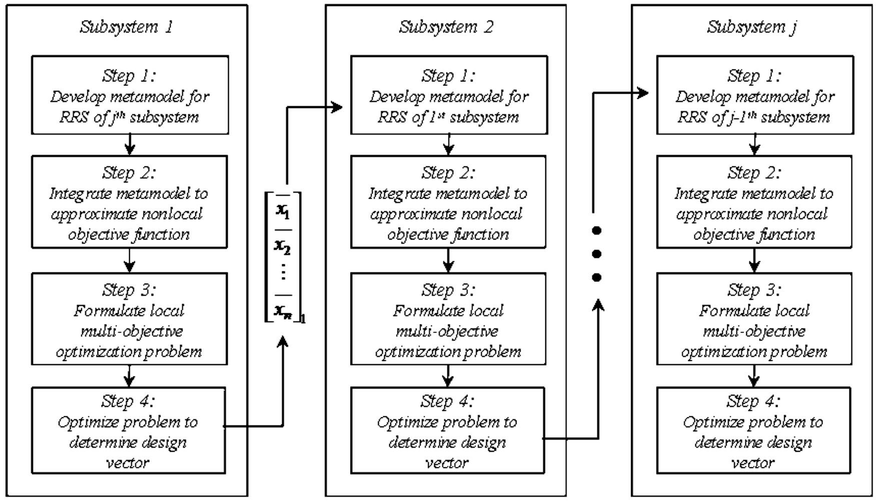

Fig. 2 Modified approximation-based decentralized design (MADD) framework.

without actually sharing objective function information. This process is seen as a potential aid to improve upon the Nash equilibrium solution and move closer toward the Pareto set, though achieving an actual Pareto point is only possible if the approximated nonlocal objective function is the true representation of the nonlocal objective function. The schematic of the MADD framework for j subsystems is shown in Fig. 2 (only stage II is shown).

The MADD framework is composed of two stages. The first stage is simply an initialization stage where the design vectors that store multiple instances of local design variable values are initialized for all subsystems. The second stage is an iterative process between the subsystems where the approximation of a nonlocal RRS, integration and summation to obtain the approximation to the nonlocal objective function, and the solution of the multi-objective optimization problem are carried out. The individual stages are discussed next.

Stage I of the MADD framework is the initialization phase where the design configuration for each subsystem is initialized. The initialization of the first subsystem in the sequential process is different from the rest of the subsystems. It is noted that any one of the several subsystems can be the first subsystem in this process. The overall solution is not impacted by which subsystem serves as the first decision maker in the MADD framework.

The initialization of the first subsystem in the sequential iterative process requires the subsystem to first solve its local optimization problem and then generate a vector of discrete design variable values selected based on a probability density function. This probability density function is centered on the local optimum value obtained by the subsystem. The generated vector is then passed to the following subsystem that optimizes its optimization problem for each entry of the communicated vector of nonlocal design variable values. Nonlocal variables are treated as equality constraints and the optimum value for the local design variables is obtained. The generated vector is then passed to the next subsystem where the same process is carried out. This is sequentially performed by all remaining subsystems.

The vector of designs generated by the first subsystem serves as a model for its own bounded rationality. This is because bounded rationality translates to the decision maker not making accurate optimal selections and one approach to model inaccurate or erroneous selections is to use a probability density function. Therefore, multiple designs that can be mistakenly selected near the true optimal solution serves as the model for the first subsystem's bounded rationality. Although this paper uses a random sampling approach to populate the vector of designs, other possible techniques for generating such designs can also be used. Design of experiments is one such possible technique that provides several different methods for sampling, for example, full/fractional factorial design, composite design, Latin hypercube sampling, etc. [42].

The communication of a vector of design variable values within subsystems is not new in decentralized design. Communicating multiple designs within a design process is broadly termed as set-based design [43,44]. Traditional set-based design approaches advocated communicating a continuous range of design variable values, where the "set" included all values between a specified upper and lower bound. This is the critical distinction between traditional set-based design methods and the approach presented here. In this work, a vector of discrete design variable values forms the communicated set.

The mathematical notation of the vector generated by the first subsystem is given in Eq. (9):

$$\bar{v}_i^j = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_n \end{bmatrix}_i^j \tag{9}$$

where $\bar{x}_1, \ldots, \bar{x}_n$ are multiple values for each design variable $x_1, x_2, \ldots, x_n$, j is the subsystem number index ($j = 1$ for the first subsystem), i is the iteration number index ($i = 0$ for the initialization stage), and $\bar{v}_i^j$ is the communicated design vector.

An important issue in implementing the MADD framework pertains to the dimensionality of the vector $\bar{v}_i^j$. The size of $\bar{v}_i^j$ directly corresponds to the number of design variables being communicated between subsystems and the order of the nonlocal RRS approximations being developed by each subsystem. As the number of variables and order of approximations increase, the number of points required to be communicated between subsystems will also increase. It is acknowledged that the current development of the MADD framework is restricted to smaller scale problems with low-order polynomial approximations (linear or quadratic) and a smaller number of design variables (5–10). Current research is investigating the dimensionality issue as the MADD framework is implemented on large scale problems [45].

At the conclusion of the first stage, the jth subsystem passes its design vector back to subsystem 1, where the second stage is invoked. A schematic of the MADD framework's second stage is shown in Fig. 2.

As mentioned previously, the process of determining the approximations of nonlocal RRS [Eq. (8)], integrating the RRS [Eq. (7)], combining the integrations to determine approximations to nonlocal objective functions [Eq. (6)], and solving the local multi-objective optimization problem comprises the second stage of the MADD framework. The individual steps of the MADD framework shown in Fig. 2 are discussed next.

Step 1. Develop a metamodel for RRS of the $(j-1)$th subsystem: At any iteration i, subsystem j possesses two pieces of information—a) a design vector $\bar{v}_i^{j-1}$ passed from subsystem $(j-1)$, and b) a local design vector $\bar{v}_{i-1}^j$ from the previous iteration (iteration $i-1$).

Because the design vector of subsystem $(j-1)$ is a reaction to the selected design of subsystem j from the previous iteration, $\bar{v}_i^{j-1}$ and $\bar{v}_{i-1}^j$ can be used to create a response surface model of subsystem $(j-1)$'s rational reaction set. This response surface model is a function that maps the local design variables of subsystem j to each of subsystem $(j-1)$'s design variables. This response surface model is mathematically represented in Eq. (10):

$$\tilde{R}^{j-1} = f\left(\bar{v}_i^{j-1}; \bar{v}_{i-1}^j, \beta\right) \tag{10}$$

where $\tilde{R}^{j-1}$ is the design variable vector of subsystem $(j-1)$, $\bar{v}_i^{j-1}$ is the design vector of subsystem $(j-1)$ at iteration i, $\bar{v}_{i-1}^j$ is the design vector of subsystem j at iteration $(i-1)$, and β is the coefficient of the approximation function.

In Eq. (10), f is the response surface model developed with $\bar{v}_{i-1}^j$ and $\bar{v}_i^{j-1}$ as the input and output vectors, respectively. Different polynomial functions can be used to fit the data to approximate the nonlocal subsystem rational reaction set. The choice of the data fitting function is not prescribed in the framework and is dependent on the data. The vector notation is used for the rational reaction set $\tilde{R}^{j-1}$ because there exists an individual function that maps the local design variables to each nonlocal design variable. It is noted that during any iteration, each subsystem develops an approximation of the objective function from the previous subsystem.

Step 2. Integrate a metamodel to approximate the nonlocal objective function: Given the approximated rational reaction set of subsystem $(j-1)$ from step 1, it is now possible to determine an approximation for the nonlocal objective functions as discussed earlier. For example, Eq. (8) is rewritten as follows:

$$R(x_1) = x_1 - \frac{a}{2} - \frac{c}{2}y_1 - \frac{d}{2}y_2 \qquad R(x_2) = x_2 - \frac{b}{2} - \frac{e}{2}y_1 - \frac{f}{2}y_2 \tag{11}$$

$R(x_1)$ and $R(x_2)$ are used to represent the derivatives of F_1 with respect to x_1 and x_2, respectively. $R(x_1)$ and $R(x_2)$ are used only for notation. Because the coefficients a, b, c, d, e, and f are obtained from approximating nonlocal RRS, comparing Eq. (11) to Eq. (7), it can be stated that Eq. (11) is the approximation to the derivative of the nonlocal objective function. Integrating this approximated derivative results in an approximation of the nonlocal objective function. The generalized expression is shown in Eq. (12):

$$\tilde{F}_i^{j-1} = \int \left(\tilde{R}^{j-1} - f\left(\bar{v}_i^{j-1}; \bar{v}_{i-1}^j, \beta\right)\right) \mathrm{d}\tilde{R}^{j-1} \tag{12}$$

It is noted that for Eq. (12), there exists an independent integral for each design variable of subsystem $(j-1)$. The mathematical representation of the objective function is formed by the addition of the individual integrals. The verification that the summation of the individual RRS integrals results in an approximation of the nonlocal objective function is provided in the Appendix of this paper.

Step 3. Formulate local multi-objective optimization problem: With the approximation to the nonlocal objective function determined, it is now possible to formulate a local multi-objective optimization problem. For this work, a simple weighted sum of the local and approximated nonlocal objective function is formulated and solved. The optimization problem in standard form is shown in Eq. (13):

$$\text{Minimize} \;\; w_1 F^j + w_2 \tilde{F}_i^{j-1} \qquad \text{s.t.}\;\; g^j(\bar{R}^j) \leq 0 \qquad \bar{R}_L^j \leq \bar{R}^j \leq \bar{R}_U^j \tag{13}$$

where w_1 is the selected weight for the local objective function, w_2 is the selected weight for the approximated nonlocal objective function, F^j is the local objective function, $\tilde{F}^{j-1}$ is the approximated objective function of subsystem $(j-1)$, g^j are the local constraints, $\bar{R}^j$ is the vector representing local design variables, $\bar{R}_L^j$ are the lower bounds of local design variables, and $\bar{R}_U^j$ are the upper bounds of local design variables.

Some notes on the multi-objective optimization problem of Eq. (13) are as follows:

1) The multi-objective optimization problem is only subject to the local constraints. This is because the nonlocal constraints are built into the approximations of the nonlocal objective functions. Specifically, because the RRS is a set of solutions that a subsystem would arrive upon by optimizing its own objective function, any metamodel built using points from the nonlocal RRS ensures satisfaction of constraints within the nonlocal subsystem. This is analogous to approximating the pseudo-objective function of the nonlocal subsystem (objective function + penalty terms).

2) It is known that the weighted sum method for multi-objective optimization problems is only useful for convex Pareto sets. (Nonconvex regions of the Pareto frontier cannot be obtained using a weighted sum method.) Other aggregation methods can be used for solving the local multi-objective optimization problem such that designs in nonconvex regions can be selected. However, only a single solution for the multi-objective optimization problem is required. The entire Pareto front for the local multi-objective optimization problem is not generated. The weighted sum approach is used in this paper for illustration purposes.

3) The choice of weights is critical as it directly relates to the level of cooperation desired by the subsystems. Setting w_1 to 1 would not add any benefit of using the MADD framework because the approximated nonlocal objective function would play no part in the optimization. Additionally, a value of 0 for w_1 translates to complete cooperation by the local subsystem. If the nonlocal subsystem chooses to not cooperate in such a case, the solution would lie on one end of the Pareto set.

4) The multi-objective optimization problem is solved for each instance of the communicated design vector.

Step 4. Optimize the problem to determine the design vector: Finally, the multi-objective optimization problem is solved for each value of the design vector, resulting in an output vector that is communicated to the next subsystem.

In this section, the MADD framework and its individual steps are discussed. The design information communication between the subsystems follows the sequential iterative process until convergence. Because a vector of designs is being passed between subsystems, using the "change in design variable values" as a measure of convergence is not practical. In the next section, a two-subsystem case study is used to illustrate the MADD framework and a new convergence criterion that uses RRS intersections is presented.

IV. MADD Framework: Case Study

In this section, a two-subsystem decentralized design problem is presented as a case study and solved using the MADD framework. Consider a decentralized design problem with two subsystems. The optimization problems and design variables for the two subsystems are given in Eqs. (14) and (15), respectively.

Subsystem 1:

Minimize

$$F_1 = 2x_1^2 + x_2^2 + 2x_1x_2 + x_1y_1 + 2x_1y_2 + x_2y_2 - x_2y_3 - 2x_1 + 3x_2 \tag{14}$$

Subsystem design variables: x_1 and x_2.

Subsystem 2:

Minimize

$$F_2 = 2y_1^2 + y_2^2 + \frac{y_3^2}{3} + y_1x_1 - y_2x_2 - 2y_3x_1 + 3y_1 + 2y_2 + 2y_3 \tag{15}$$

Subsystem design variables: y_1, y_2, and y_3.

The decentralized design problem is first solved using a standard sequential iterative approach. The converged solution of the iterative process lies at the intersection of the individual subsystems' RRS.

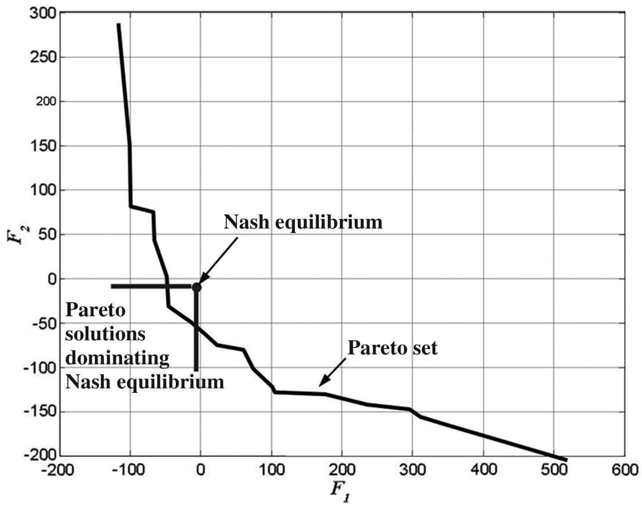

Fig. 3 Pareto set and Nash equilibrium for two-subsystem decentralized design.

Because the objective functions of the two subsystems are unconstrained, the RRS for each subsystem is obtained by taking the partial derivatives of the objective function with respect to the locally controlled design variables and setting them to zero. The resulting system of equations is solved to obtain the Nash equilibrium solution. The design variable and corresponding objective function values for the Nash equilibrium are given in Eq. (16).

$$\begin{matrix}\text{Subsystem 1} \\ \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 2.1717 \\ -1.1313 \end{Bmatrix} \\ F_1 = -5.7988\end{matrix} \qquad \begin{matrix}\text{Subsystem 2} \\ \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} -1.2929 \\ -1.5657 \\ 3.5152 \end{Bmatrix} \\ F_2 = -9.9138\end{matrix} \tag{16}$$

Using a multi-objective genetic algorithm (MOGA) [46–50], the Pareto set for the problem is obtained. The Pareto set along with the Nash equilibrium is shown in Fig. 3.

The curve in Fig. 3 represents the Pareto frontier obtained using the MOGA when viewing the objective functions of the two subsystems as a multi-objective optimization problem. As seen from the figure, the Nash equilibrium is dominated by a number of Pareto solutions (solutions within the right angle region).

In the next section, the MADD framework is implemented to seek solutions that are an improvement over the traditionally obtained Nash equilibrium solutions.

A. Case Study: Implementing MADD Framework

In this section, the MADD framework introduced in Sec. III is used to solve the two-subsystem case study. Detailed descriptions for the implementation of each step are not provided but rather key formulations and results are presented in this section.

One important formulation and contribution within the MADD framework is the determination of the nonlocal objective function approximation. The determination of this approximation is presented next for both subsystems.

Subsystem 1:

The task of subsystem 1 is to minimize F_1 which is a function of x_1, x_2, y_1, y_2, and y_3. The design variables controlled by subsystem 1 are x_1 and x_2. Therefore, subsystem 1 develops approximations of y_1, y_2, and y_3 as a function of its locally controlled design variables x_1 and x_2. Because the objective functions are quadratic, the approximations to the RRS are chosen to be linear models. The generalized forms of the approximated RRS are given in Eq. (17):

$$y_1 = a_1 x_1 + a_2 x_2 + a_3 \qquad y_2 = b_1 x_1 + b_2 x_2 + b_3 \qquad y_3 = c_1 x_1 + c_2 x_2 + c_3 \tag{17}$$

Next, the RRSs are individually integrated and the integrals are summed to determine an approximation of the nonlocal objective functions. The approximated objective functions of subsystem 2 are shown in Eq. (18):

$$\begin{aligned}\tilde{F}_2 &= \int (y_1 - (a_1x_1 + a_2x_2 + a_3))\,\mathrm{d}y_1 \\ &+ \int (y_2 - (b_1x_1 + b_2x_2 + b_3))\,\mathrm{d}y_2 \\ &+ \int (y_3 - (c_1x_1 + c_2x_2 + c_3))\,\mathrm{d}y_3 = \frac{y_1^2}{2} + \frac{y_2^2}{2} + \frac{y_3^2}{2} \\ &- (a_1x_1 + a_2x_2 + a_3)y_1 - (b_1x_1 + b_2x_2 + b_3)y_2 \\ &- (c_1x_1 + c_2x_2 + c_3)y_3\end{aligned} \tag{18}$$

Equation (17) is used in conjunction with F_1 [Eq. (14)] to formulate the local multi-objective optimization problem (stage II: step 3 of the MADD framework). The weighted sum formulation solved by subsystem 1 during each iteration of the sequential process is shown in Eq. (19):

$$F_1^{\text{new}} = w_1^1 F_1 + w_2^1 \tilde{F}_2 \tag{19}$$

where F_1^{new} is the new objective function minimized by subsystem 1 during each iteration. The superscripts on the weight variables indicate that these are the weights set by subsystem 1. The formulations for subsystem 2 are given next.

Subsystem 2:

Analogous to subsystem 1, subsystem 2 approximates the RRS of subsystem 1 using the communicated design variables. The generalized form of subsystem 1 RRS approximated by subsystem 2 is shown in Eq. (20):

$$x_1 = d_1y_1 + d_2y_2 + d_3y_3 + d_4 \qquad x_2 = e_1y_1 + e_2y_2 + e_3y_3 + e_4 \tag{20}$$

The integration and summation of the approximated RRSs forms the approximation of the objective function of subsystem 2. This is shown in Eq. (21).

$$\begin{aligned}\tilde{F}_1 &= \int (x_1 - (d_1y_1 + d_2y_2 + d_3y_3 + d_4))\,\mathrm{d}x_1 \\ &+ \int (x_2 - (e_1y_1 + e_2y_2 + e_3y_3 + e_4))\,\mathrm{d}x_2 \\ &= \frac{x_1^2}{2} + \frac{x_2^2}{2} - (d_1y_1 + d_2y_2 + d_3y_3 + d_4)x_1 \\ &- (e_1y_1 + e_2y_2 + e_3y_3 + e_4)x_2\end{aligned} \tag{21}$$

The modified objective function minimized by subsystem 2 is the weighted sum of Eqs. (15) and (21) and is shown in Eq. (22):

$$F_2^{\text{new}} = w_1^2 \tilde{F}_1 + w_2^2 F_2 \tag{22}$$

Equations (19) and (22) are solved in a sequential iterative process until both subsystems achieve convergence.

To determine the final converged solution, the intersection of the RRS's (the Nash equilibrium) is found. Because the modified objective function for each subsystem is unconstrained, the RRS for each subsystem can be obtained by differentiating Eqs. (19) and (22) with respect to their individually controlled design variables. These derivatives are then set equal to zero and rearranged to determine the optimal values of local design variables as a function of the nonlocal design variables, the approximation parameters, and the objective weights. The new RRS are shown in Eq. (23) for subsystem 1 and Eq. (24) for subsystem 2.

Subsystem 1:

$$x_1 = \frac{1}{4w_1^1}[w_2^1(a_1y_1 + b_1y_2 + c_1y_3) - w_1^1(2x_2 + y_1 + 2y_2 - 2)]$$

$$x_2 = \frac{1}{2w_1^1}[w_2^1(a_2y_1 + b_2y_2 + c_2y_3) - w_1^1(2x_1 + y_2 - y_3 + 3)] \tag{23}$$

Subsystem 2:

$$y_1 = \frac{1}{4w_2^2}\left[w_1^2(d_1x_1 + e_1x_2) - w_2^2(x_1 + 3)\right]$$

$$y_2 = \frac{1}{2w_2^2}\left[w_1^2(d_2x_1 + e_2x_2) - w_2^2(2 - x_2)\right] \tag{24}$$

$$y_3 = \frac{3}{2w_2^2}\left[w_1^2(d_3x_1 + e_3x_2) - w_2^2(2 - 2x_1)\right]$$

Therefore, the final solution for the two subsystems can be determined from Eqs. (23) and (24) after the coefficients for the nonlocal RRS of Eqs. (17) and (20) are determined.

The sequential iterative process continues between the two subsystems until they converge. Before converged results for various objective weights are presented, the convergence criterion employed for this problem is discussed.

Convergence Criterion: In traditional multidisciplinary design optimization, changes in objective function values are commonly used as the primary convergence criterion. However, in the implementation of the MADD framework, each subsystem combines an approximation of the nonlocal subsystem's objective function with its own objective function before optimization. Because the objective function optimized at each iteration is modified, the use of changes in objective function values as a convergence criterion is not practical.

For the problem presented in this section, a convergence criterion is developed based on the rational reaction sets. The new, individual subsystem RRS [shown in Eqs. (23) and (24)] continuously changes every iteration based on the approximations of the nonlocal RRS. Therefore, the classical definition of convergence (intersection of RRS) cannot be employed in this situation.

To overcome this problem, convergence is sought by determining the "average" RRS intersection. The convergence criterion defined here states that the iterative process has converged if the average intersection of the RRS does not change by some set limit over several iterations. Mathematically, this is represented in Eq. (25):

$$\bar{\mu}_i^* - \bar{\mu}_{i-1}^* \leq \delta \qquad \forall\ i = 1, 2, \ldots, n \tag{25}$$

where

$$\bar{\mu}_i^* = \frac{(\bar{\mu}_{i-1}^* * i) + \bar{x}_i^*}{(i + 1)}$$

where $\bar{\mu}_i^*$ is the average intersection of the RRS at the ith iteration, $\bar{x}_i^*$ is the variable value at the RRS intersection for the ith iteration, δ is the allowable tolerance, and n the number of iterations over which the average intersection does not change by more than the allowable tolerance.

For the case study problem, the intersection of the two subsystems RRS is obtained by solving the system of linear equations given in Eqs. (23) and (24). This system of linear equations is evaluated at every iteration using the preset values for objective weights w_1^1, w_2^1, w_1^2, and w_2^2. The obtained values of the design variables are compared to the previous iterations to determine convergence. As an illustration, one solution obtained using the MADD framework is presented using the weights for the objective functions as

$$w_1^1 = 0.9 \qquad w_1^2 = 0.1$$
$$w_2^1 = 0.1 \qquad w_2^2 = 0.9$$

Because the MADD framework is an inherently stochastic process with designs selected from a normal distribution by subsystem 1 during stage I, it is essential to determine the expected converged solution of the MADD framework. The expected solution is determined by running the simulation several times and determining the expected value. Table 1 lists the objective function and design variable values of the expected converged solution and compares them to the Nash equilibrium.

From Table 1, it is seen that the expected solution of the MADD framework dominates the Nash equilibrium solution for both objectives. The solution is not Pareto optimal but an improvement over the Nash equilibrium.

As discussed in Sec. III, the MADD solution is subject to the setting of several parameters. The parameter values set for the MADD framework solution of the case study problem are shown in Table 2.

Among the parameters presented in Table 2, the important parameters set within the MADD framework are the weights on the local and approximated nonlocal objective functions, the number of design variables communicated during any iteration, and the initial standard deviation to model bounded rationality of the first designer. These are considered the most important parameters as they play a direct role in the formulation and solution of the modified problems of the MADD framework. Therefore, it is important to study the effect of change in these parameters on the final solution. This study is presented in the next section.

B. MADD Framework: Parameter Study

1. Number of Designs Communicated Between Subsystems at Every Iteration (Stage I of the MADD Framework)

To study the effect of the number of communicated designs on the converged solution, the MADD framework simulation is run for different numbers of communicated designs, namely, 2, 5, 10, 20, and 30 designs communicated during every iteration. As seen from Fig. 4, the expected solution for F_1 and F_2 changes significantly when going from two to five communicated designs. However, beyond five, the solution does not change as the number of communicated designs increases.

The cause for the imperfect solution when two communicated designs are used is because of the limitations on the number of coefficients in the RRS approximation and the form of the metamodeling polynomial. For the case study problem, the RRS approximations are linear with three unknowns. Therefore, at the minimum, three data points would be required to determine one possible RRS approximation. Beyond three, if higher order polynomials are being used for metamodeling, more data points would improve the fidelity of the fit. Hence, for the case study problem, it is concluded that when using a sufficiently large number of designs communicated between the subsystems, the converged solution is independent of the number of communicated designs.

Table 1 Comparison of MADD framework and Nash solutions

	Expected MADD solution	Nash equilibrium
F_1	−6.0128	−5.7988
F_2	−10.713	−9.9138
x_1	2.2992	2.1717
x_2	−0.965	−1.1313
y_1	−1.4168	−1.2929
y_2	−1.6121	−1.5657
y_3	3.6516	3.5152

Table 2 Parameter values for MADD framework implementation

No. of designs communicated every iteration	5
Initial standard deviation to model bounded rationality in stage I	0.01
No. of iterations over which RRS intersections do not change by an allowable tolerance (used for convergence)	10
Allowable tolerance	0.8

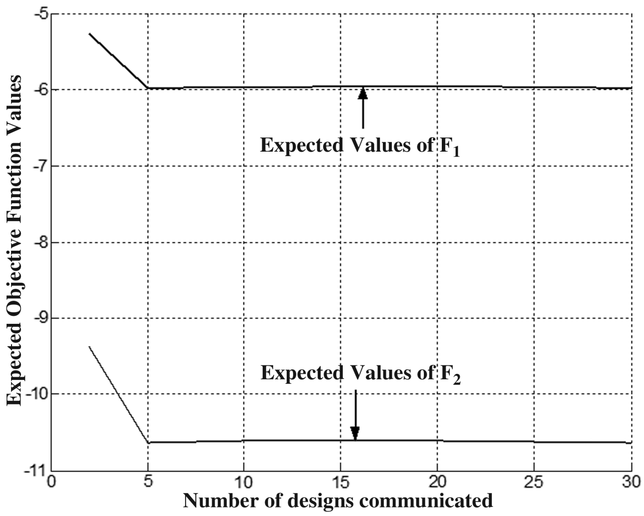

Fig. 4 Expected objective function values vs number of designs communicated.

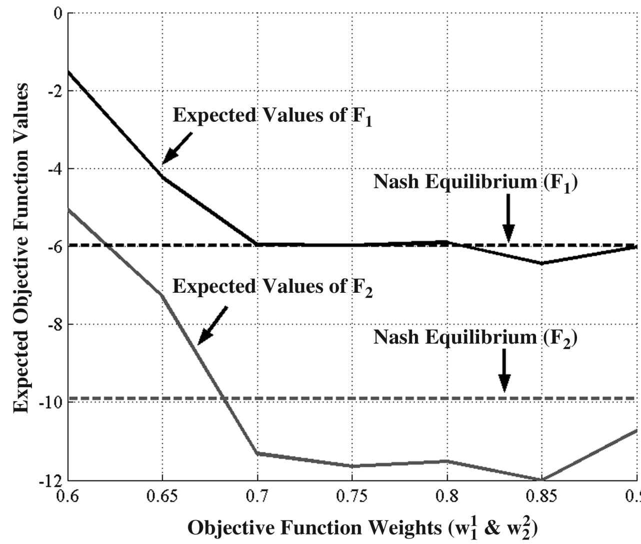

Fig. 6 Expected objective function values vs objective function weights.

2. Initial Standard Deviation Set by the First Subsystem (Stage I of the MADD Framework)

To study the effect of the standard deviation used to model bounded rational choices of the first subsystem on the final solution, five values of the initial standard deviation are set and the expected values of the convergent solutions are determined. The values for σ are 0.001, 0.01, 0.1, 1, and 2. For each of these values, the MADD framework is run several times and the expected objective function solution is obtained. The variation in the expected values of the objective functions as a function of the changing σ values is shown in Fig. 5.

In Fig. 5, the top line represents F_1, as a function of σ, while the lower line represents F_2. The simulation is run by setting the number of communicated designs constant at 5 and the objective weights as $w_1^1 = 0.9$, $w_2^1 = 0.1$, $w_1^2 = 0.1$, and $w_2^2 = 0.9$.

As seen in Fig. 5, the expected value of the objective function solution changes slightly as σ is increased but beyond a small value, it does not change significantly. This implies that σ does not have a significant impact on the final solution. It is important to note that $\sigma = 0$ should not be used as this would imply that a vector of exactly the same designs is communicated during every iteration, resulting in erroneous fitting parameters in the second stage of the MADD framework. Moreover, this would reduce the problem to the regular sequential iterative case resulting in convergence to the original Nash equilibrium.

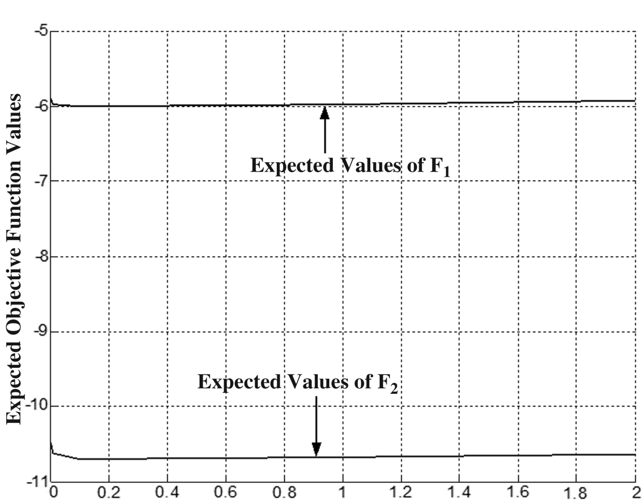

Fig. 5 Expected objective function values vs standard deviation.

3. Weights for Objective Functions (Stage II of the MADD Framework)

For the objective weight variables, the following relationships hold: $w_1^1 + w_2^1 = 1$ and $w_1^2 + w_2^2 = 1$. As part of this study, simulations of the MADD framework are performed by selecting different values for w_1^1 and w_2^2. The different values for the weights selected are

$$w_1^1\!: \; = 0.9, 0.85, 0.8, 0.75, 0.7, 0.65, 0.6$$
$$w_2^2\!: \; = 0.9, 0.85, 0.8, 0.75, 0.7, 0.65, 0.6$$

Both objective function weights are simultaneously incremented for each simulation run. The selected weights are only reduced to 0.6 because decreasing both weights beyond 0.6 simultaneously results in solutions that either do not converge or take a long time to converge. This is because for weight values less than 0.6, the new local objective functions apply too high of an importance to the approximated nonlocal objective function. Any errors in modeling the nonlocal objective function propagate and get emphasized by the higher weight value creating convergence challenges.

The remaining parameters of the MADD framework are set as follows: the number of designs communicated is set to 10 and the standard deviation is set to 0.01. The MADD framework is run multiple times and the average solution is determined. Figure 6 presents results of these simulations.

In Fig. 6, the two curves shown in solid black and gray lines represent the expected values of F_1 and F_2, respectively, as w_1^1 and w_2^2 are varied. The dashed black and gray horizontal lines correspond to the Nash equilibrium solution for the two objective functions, respectively. As seen in Fig. 6, the MADD framework solution is an improvement over the Nash equilibrium solution of F_2 for all values of w_2^2 greater than 0.7. For F_1, values of w_1^1 greater than 0.75 result in an improvement over the Nash solution. Additionally, for $w_1^1 = 0.75$, the solution is a very slight improvement over the Nash equilibrium.

However, for both objective functions the expected solutions are significantly inferior to the Nash equilibrium when w_1^1 and w_2^2 are less than 0.7. This is because both subsystems are favoring solutions that are better for the approximations of the nonlocal objective, thus collectively declining the quality of the solution. In the next section, conclusions of this parametric study and the overall paper are presented.

V. Conclusions

In this paper, the assumption of a designer's ability to make perfectly rational choices within decentralized design systems is questioned. The notion of bounded rationality is introduced and a new approach for design communication between decentralized subsystems using bounded rationality is proposed. The unique contribution of this work is the ability to improve upon the Nash equilibrium solution without actually sharing an objective function or gradient information. This improvement is achieved by developing approximations of nonlocal objective functions through approximations of the nonlocal RRS. Determining approximations of the RRS as opposed to approximations of the objective functions is the novel contribution of this research to decentralized design.

For the parameter study, it is seen that the number of communicated designs impacts the final solution only if a very small number of designs is used. Additionally, the standard deviation used to model bounded rational choices of the first subsystem has very little impact on the final solution whereas the objective weights have significant impact on the final converged solution. Though these results are specific for the case study problem, the discussion has sought to provide the reader with an understanding of a general problem.

Another point of discussion pertains to the computational complexity of implementing the MADD framework. As the number of subsystems and complexity of subsystems' optimization problem increases, the computational cost of implementing the MADD framework would increase significantly. However, in most real world engineering problems, despite the increase in computational infrastructure, metamodels and approximations are still developed and used in place of the original computationally expensive analysis programs [51,52]. These approximations are built over the input parameters and design variables that have the most significant impact on the outputs. Therefore, the MADD framework can be used within decentralized design problems to approximate nonlocal RRS corresponding to the nonlocal subsystem's metamodel.

Given that the MADD framework can be used to approximate nonlocal subsystem's metamodels, the number of design variables being communicated between subsystems also adds to the computational complexity of the framework. For example, assuming that nonlocal subsystems are using quadratic approximations as their metamodels, the number of variables that make up the metamodel is given by the relationship in Eq. (26):

$$N_c = \frac{n^2}{2} + \frac{3n}{2} + 1 \tag{26}$$

N_c corresponds to the number of coefficients in a quadratic model and n is the number of variable inputs to the metamodel. For $n = 2$, the number of coefficients is 6. However, real engineering problems usually have a large number of variables. For $n = 100$, the number of coefficients is 5151. For each subsystem to approximate such a large number of nonlocal subsystem coefficients significantly increases the size of the vector given in Eq. (9) as well as the number of optimizations to be performed by each subsystem. Therefore, the MADD framework is restricted to problems with smaller numbers of variables (5–10) being communicated between subsystems.

Although the application of bounded rationality to decentralized design and multidisciplinary design optimization is the novel contribution of this paper, it is acknowledged that it is the first step in solving the real decentralized design problem comprising several subsystems each optimizing complex analyses. The following are some of the identified areas of future work for the research presented in this paper:

1) The MADD framework needs to be implemented for three or more subsystems with complex analyses such as finite element or computational fluid dynamics (CFD) codes. As mentioned in the discussion on stage I of the MADD framework (Sec. III), the application of MADD framework to large scale problems will require investigation into the dimensionality of the design variable vectors communicated between subsystems [Eq. (9)].

2) The presented methodology proposes to approximate nonlocal subsystem problems as unconstrained objectives under the assumption that the communicated points between subsystems are from the nonlocal RRS. This assumption ensures the communicated points are optimal within the nonlocal subsystem domain. However, for large scale problems, a more direct approach to handling nonlocal constraints is required, which directly incorporates the Karush-Kuhn Tucker (KKT) conditions for optimality of constrained problems. One possibility of incorporating the KKT conditions, for example, would be through the use of a Lagrangian within the model presented in Eq. (8).

3) Although this paper incorporates models for bounded rationality only for the first subsystem, a significant step forward for this research is to consider bounded rationality for *all* subsystems within the decentralized design process. This would increase the complexity of the overall problem and introduce significant computational challenges.

4) This paper uses a simple random sampling model for bounded rationality. A significant avenue for future work is to investigate and develop more complex models for bounded rationality and incorporate these models within the MADD framework.

5) The sufficient and necessary conditions for convergence of the MADD framework need to be developed.

Appendix

To mathematically verify that the summation of individual RRS integrals results in an approximation of the nonlocal objective function shown in Eq. (8), consider a subsystem B with objective function F_2 that is being approximated by a subsystem A. Let the design variables controlled by B be x_2, y_2, and z_2. Additionally, let F_2 depend on x_1, y_1, and z_1, the design variables controlled by subsystem A. From Eq. (10), subsystem A has the following approximations:

$$x_2 = f^x(x_1, y_1, z_1) \qquad y_2 = f^y(x_1, y_1, z_1) \qquad z_2 = f^z(x_1, y_1, z_1) \tag{A1}$$

The variables f^x, f^y, and f^z represent response surface equations for the three design variables. As stated earlier, the difference between the right- and left-hand sides of Eq. (2) is the approximation of the partial derivatives of the nonlocal objective function. Therefore, rewriting Eq. (A1), we get

$$\frac{\partial F_2}{\partial x_2} = x_2 - f^x(x_1, y_1, z_1) \qquad \frac{\partial F_2}{\partial y_2} = y_2 - f^y(x_1, y_1, z_1)$$
$$\frac{\partial F_2}{\partial z_2} = z_2 - f^z(x_1, y_1, z_1) \tag{A2}$$

Now, given the approximations to derivatives of F_2, the approximation to F_2 needs to be determined. For a function of multiple variables such as F_2, the exact differential can be written as shown in Eq. (A3),

$$\mathrm{d}F_2 = \frac{\partial F_2}{\partial x_2}\,\mathrm{d}x_2 + \frac{\partial F_2}{\partial y_2}\,\mathrm{d}y_2 + \frac{\partial F_2}{\partial z_2}\,\mathrm{d}z_2 \tag{A3}$$

The exact differential of F_2 does not include terms for x_1, y_1, and z_1 since the partial derivatives of F_2 with respect to these variables are zero. Now, let a function u_2 be defined such that

$$u_2 = \int \frac{\partial F_2}{\partial x_2}\,\mathrm{d}x_2 \tag{A4}$$

Therefore,

$$u_2 = \int (x_2 - f^x(x_1, y_1, z_1))\,\mathrm{d}x_2$$
$$u_2 = \frac{x_2^2}{2} - x_2 f^x(x_1, y_1, z_1) + g_2(y_2, z_2) \tag{A5}$$

The constant of integration in Eq. (A5) is a function of y_2 and z_2 represented as g_2. The next task is to determine g_2.

The exact differential of u_2 can be written as follows:

$$\mathrm{d}u_2 = \frac{\partial u_2}{\partial x_2}\,\mathrm{d}x_2 + \frac{\partial u_2}{\partial y_2}\,\mathrm{d}y_2 + \frac{\partial u_2}{\partial z_2}\,\mathrm{d}z_2$$
$$\Rightarrow \mathrm{d}u_2 = \frac{\partial F_2}{\partial x_2}\,\mathrm{d}x_2 + \frac{\partial g_2}{\partial y_2}\,\mathrm{d}y_2 + \frac{\partial g_2}{\partial z_2}\,\mathrm{d}z_2 \tag{A6}$$

For u_2 to be equal to F_2, it is required that

$$\frac{\partial u_2}{\partial y_2} = \frac{\partial F_2}{\partial y_2} \quad \text{and} \quad \frac{\partial u_2}{\partial z_2} = \frac{\partial F_2}{\partial z_2} \tag{A7}$$

Therefore, from Eqs. (A5) and (A7) we get

$$\frac{\partial g_2}{\partial y_2} = \frac{\partial F_2}{\partial y_2} \quad \text{and} \quad \frac{\partial g_2}{\partial z_2} = \frac{\partial F_2}{\partial z_2} \tag{A8}$$

Now, the exact differential of g_2 is

$$\mathrm{d}\,g_2 = \frac{\partial g_2}{\partial y_2}\,\mathrm{d}y_2 + \frac{\partial g_2}{\partial z_2}\,\mathrm{d}z_2 \tag{A9}$$

Let

$$v_2 = \int \frac{\partial g_2}{\partial y_2}\,\mathrm{d}y_2$$

Therefore,

$$v_2 = \int \frac{\partial g_2}{\partial y_2}\,\mathrm{d}y_2 = \int \frac{\partial F_2}{\partial y_2}\,\mathrm{d}y_2 = \int (y_2 - f^y(x_1, y_1, z_1)\,\mathrm{d}y_2$$
$$= \frac{y_2^2}{2} - y_2 f^y(x_1, y_1, z_1) + h_2(z) \tag{A10}$$

For v_2 to equal g_2,

$$\frac{\partial v_2}{\partial z_2} = \frac{\partial g_2}{\partial z_2}$$

needs to be true. From Eqs. (A1), (A8), and (A10), we get

$$\frac{\partial F_2}{\partial z_2} = \frac{\partial g_2}{\partial z_2} = \frac{\partial v_2}{\partial z_2} = \frac{\mathrm{d}h_2}{\mathrm{d}z_2} = z_2 - f^z(x_1, y_1, z_1) \tag{A11}$$

Because Eq. (A11) is an ordinary differential equation, it can be integrated to determine $h_2(z)$. This is shown in Eq. (A12):

$$\frac{\mathrm{d}h_2}{\mathrm{d}z_2} = z_2 - f^z(x_1, y_1, z_1) \Rightarrow h_2(z) = \frac{z_2^2}{2} - z_2 f^z(x_1, y_1, z_1) + c \tag{A12}$$

The constant of integration c is assumed to be zero. Combining Eqs. (A5), (A10), and (A12), the functional notation for the objective function of subsystem B as approximated by subsystem A is given in Eq. (A13):

$$\tilde{F}_2 = \frac{x_2^2}{2} + \frac{y_2^2}{2} + \frac{z_2^2}{2} - x_2 f^x(x_1, y_1, z_1)$$
$$- y_2 f^y(x_1, y_1, z_1) - z_2 f^z(x_1, y_1, z_1) \tag{A13}$$

Thus, an approximation of the nonlocal objective function can be obtained by integrating the approximation to the nonlocal RRS. The function in Eq. (A13) is for three local and three nonlocal design variables but can be extended to any number of design variables. The MADD framework models all nonlocal objective functions similar to the one shown in Eq. (A13).

Acknowledgments

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